



**Faculty of Graduate Studies**

**Application of Mathematics and Game Theory to  
Industrial Organization**

*By*

**Abdelrahim Said Mousa**

**Supervisor:**

**Prof. Mohammad Saleh**

Birzeit, Palestine

Spring, 2009



**Faculty of Graduate Studies**

**Application of Mathematics and Game Theory to  
Industrial Organization**

*By*

**Abdelrahim Said Mousa**

**Members of the Examination Committee:**

- **Prof. Mohammad Saleh (Chairman of the Committee)**
- **Dr. Marwan Aloqayle (Chairman of Math. Department)**
- **Dr. Tahseen Mughrabi (Al-Quds University)**

**Date of Defense: May 14, 2009**

**Birzeit, Palestine**

**Spring, 2009**

**This thesis was submitted in partial fulfillment of the requirements for the Master Degree in Mathematics from the faculty of Graduate Studies at Birzeit University, Palestine.**

# **Application of Mathematics and Game Theory To Industrial Organization**

*Prepared By*

*Abdelrahim Said Mousa*

This thesis was prepared under the main supervision of Professor Mohammad Saleh, and it has been approved by all members of the examination committee.

**Members:**

**Signature:**

- **Prof. Mohammad Saleh (Chairman of the committee)** .....
- **Dr. Marwan Aloqayle** .....
- **Dr. Tahseen Mughrabi** .....

**The finding, interpretations and the conclusions expressed in this study do not necessarily express the views of Birzeit University, the views of the individual member of the MSc committee or the views of their respective.**

## **Dedication**

I dedicate this thesis to my family, mainly:

To my mother "Jameela" for her sincere endeavors.

To my father "Sa'id" for his useful advices.

To my brother "Mohammad" for his incessant encouragements.

To my wife "Rasha" for her endless support.

To my love daughter "Jameela".

## **Acknowledgments**

I am greatly indebted to the kind assistance and to the excellent comments of Professor Mohammad Saleh, my main supervisor, for the enthusiasm and inspiration, who was always there when I needed him and who really lead to important improvements for his continuous supervision to my thesis. I appreciate his advanced situation which is always supporting and granting me a lot of opportunities.

I would also like to exploit this opportunity to thank Dr Marwan Aloqayle and Dr Tahseen Mughrabi for their valuable comments, useful suggestions and good recommendations to my thesis.

I wish to thank all those who helped me, especially Dr. Alberto Pinto and Dr. Rosa Branca, who are patiently supported me by choosing this field of study.

# Table of Contents

Project-Members.....	I
Dedication .....	II
Acknowledgments .....	III
Table of Contents .....	IV
Abstract .....	2
<b>Chapter One: Introduction</b>	
1.1    Introduction .....	4
<b>Chapter Two: History</b>	
2.1    History .....	8
2.2    Study Site .....	10
2.3    Game Theory .....	12
2.4    Prisoners' Dilemma .....	15
<b>Chapter Three: Literature Review</b>	
3.1    Rationality (Herbert Simon 1955).....	18
3.2    Incorporating Fairness into Game Theory and Economics .....	28
3.3    Fairness Model (Rabin 1993) .....	32
3.4    Bounded Rationality .....	45
3.5    Bounded Rationality in Industrial Organization (Glenn Ellison 2006).....	48

## **Chapter Four: Generalized Models for Bounded Rationality**

4.1	Introduction .....	55
4.2	Rational Model (I) .....	56
4.3	Rational Model (II) .....	63
4.4	Rational Model (III) .....	70
4.5	Bounded Rationality in Rational Models .....	73

## **Chapter Five: Conclusions and Suggestions**

5.1	Conclusions and Suggestions .....	84
5.2	References .....	86

## المخلص

تهدف هذه الرسالة الى تطبيق الرياضيات و نظرية الألعاب في التنظيم الصناعي، حيث يوجد محوران مختلفان في الرسالة، الأول: مقدمة للرشد المحدود في نماذج التنظيم الصناعي، حيث ما زال التركيز في أدبيات التنظيم الصناعي على الفرض الذي ينص بأن الوكلاء "الزبائن" يتطلعون دائما في الحصول على أكبر قدر من الأرباح. يوجد مجال جديد في أدب التنظيم الصناعي و هو تصرف التنظيم الصناعي الذي بدوره يطرح طرق مختلفة للعزول عن فرض العقلانية التامة أو الحصول على أكبر قدر من الأرباح.

أما المحور الثاني فهو عمل اطار جديد لتحر دور " Rule of Thumb Approach " على تصرف المستهلك من جهة، و تصرف الوكلاء من جهة أخرى. في هذا المحور عملت على بناء ثلاثة نماذج لتعميم نموذج "Ellison" في الاختيار العقلاني و هي:

1. النموذج العقلاني الأول: وفيه يوجد منتج جديد واحد، و هذا النموذج هو تعميم لمسلك "Rule of Thumb Approach".
2. النموذج العقلاني الثاني: وفيه يوجد أكثر من منتج جديد واحد في ظل شركة واحدة.
3. النموذج العقلاني الثالث: وفيه يوجد أكثر من منتج جديد واحد في ظل تنافس بين الشركات.



## **Abstract**

This research project aims to apply mathematics and game theory to Industrial Organization. In this research I will follow two different approaches: The first one is related to the introduction of bounded rationality in industrial organization (IO) models. The industrial organization literature has hitherto focused on the assumption that agents are profit/utility maximizers. A new strand in the industrial organization literature, behavioral industrial organization, addresses ways to depart from the assumption of full rationality and utility maximizing agents.

The second approach proposes new framework to investigate the effects of rule of thumb approach on both consumer's behavior and firm's behavior. In this approach we construct three rational models that generalize the Ellison's model for rational choice: (i) Rational Model (I) with one new product, this model is generalization of rule of thumb approach (ii) Rational Model (II) with more than one new product within one firm and (iii) Rational Model (III) with more than one new product within competitive firms.

## **Chapter One: Introduction**

### 1.1 Introduction

## **Introduction**

Industrial organization encompasses topics such as market structure, price dispersion, and how firms compete by choosing prices, quantities, research and development levels (R&D levels), product quality and other product characteristics. Broadly, in this research we will follow two different approaches.

The first one is related to the introduction of bounded rationality in industrial organization (IO) models. For a long time, the industrial organization literature has still focused on the assumption that agents are profit/utility maximizers. (Fully Rational).

A new strand in the industrial organization literature, behavioral industrial organization, addresses ways to depart from the assumption of full rationality, utility maximizing agents. A nice overview of this new approach and its predecessors is Ellison (2005). Behavioral industrial organization is then the application of insights from psychology and then deviations from the behavior of Homo Economics (a selfish and utility-maximizing, unboundedly-rational agent) to topics that belong to industrial organization.

The deviations from fully-rational decision making are classified as behavioral. It can come in principle from any of the economic agents involved in the market. Recent research, however, usually attributes the irrational behavior to consumers and not to firms. Thus, one of this research project's goals is to investigate the role of boundedly rational consumers on firms pricing decisions.

The recent literature on competitive price discrimination, for instance, (e.g. Stole (2006), Armstrong (2006)) has assumed that consumers take their buying decisions in a purely rational way. A standard assumption is that consumers are only interested in their own self and have no regard for others.

However, in reality, people evidence regards for others and is concerned about relative payoffs. Reactions to discount policies, issues of fairness, and other factors should alter the consumers buying decisions.

For example, people care about the fairness of short-term pricing strategies of firms (Kahneman, Knetsch and Thaler (1986)). Rabin (1993) proposed a model that incorporates fairness in two-person normal form games.

His formulation becomes intractable in n-person games. Based on people concern for relative payoffs, Fehr and Schmidt (1999) proposed an alternative model of fairness, where an individual draws utility from his own payoff but also some disutility from the inequities in payoff. In contrast, to the traditional rationalist approach, this project aims to study different forms of rational models in markets where consumers display behavioral reactions to the firm's pricing strategies.

The second approach proposes new framework to investigate the effects of rule of thumb approach on both consumer's behavior and firm's behavior. Basically, there are three rational frameworks: (i) with one new product (ii) more than one new products with one firm and (iii) more than one new products within competitive firms. The second framework has received much more attention than the former.

## **Chapter Two: History**

- 2.1 History
- 2.2 Study Site
- 2.3 Game Theory
- 2.4 Prisoners' Dilemma

## History

It is not clear exactly at what time the bounded rationality in industrial organization began. But during 30 years from 1920 to 1950, firms think either to do models in order to maximize their profit or by depending (adopting) behavioral models. Bounded rationality certified new developments during 1950s).

In case of rationality, Simon (1955) wrote that if the global rational of economic man can be replaced by rational behavior man in order to make him applicable by economic information from one side, and on other side to reduce his capacity of complex calculation. In 1982, Simon suggested “satisfying models” instead of “rational models”. He identified bounded rational in order to satisfy in stead of maximize (full rational decision).

In 1994, Tanga Mc Daniel, E. Elisabet Rutstorm and Melonie William comment and added on Rabin Review (1993). They show how incomplete information weakens the case for fairness and strengthens the case for individual rationality in many games. They do this based on predictions and experimental tests of the game of Chicken.

They found that the distinction between fairness and altruism conceptually useful. They pointed two important assumptions that are necessary when testing fairness vs. rationality. These are the assumptions of complete information beliefs and mutually exclusive behaviors. When these assumptions are violated, then it is very difficult to confirm any systematic impact on behavior from subjective values based on fairness.

In Ellison (2006) "Bounded Rationality in Industrial Organization", he indicated that for all rational models, consumer apply or enter in a rule - of - thumb approach in order to derive the optimal behavior as an optimal solution to the complex game in their mind.

Some of recent literature like "Hendricks" said that the first two kinds of bounded rationality in Ellison article (2006) " rule - of - thumb approach and cognitive is costly to make decision" are closely related to each other, and both kinds have long tradition in IO, but the last one (Agent who exhibits behavioral biases) is new approach. Kenneth Hendricks comments on Ellison literature is that rule - of - thumb approach are compliment to the rational approach.

However, most articles that explain rationality or bounded rationality for industrial organization face the problems and the difficulties to derive the equilibrium strategy in rational models (equilibrium strategy in rational models is optimal behavior), because the behavior is also related to psychological issues depends on the satisfaction of players which vary from one to another.

So, in this research I will derive three rational models, in which studying the effect of the future price expectation as an external factor is possible to do bounded rational choices for the selection choices, rather than fairness, and to compare this new manner with the usual case. The new derivation will be done by using rational way for humane calculations.



## Study Site

- **1920 - 1950 :**

During this period of time, bounded rationality certified new developments and firms began thinking to do models in order to maximize their profits depending on behavioral models.

- **1955:**

The developments here are in rationality, Simon talked about rational choice, and he suggested an idea which is: if we can replace the global rational man to rational behavioral man, because the last one has limited capacity for calculation.

- **1973**

Joskous (1973) estimated the profit model as an example of behavioral to industrial organization. The profit model has the ability to estimate firm's profit. He discussed the case when this model changes to estimate the behavioral in some conditions.

- **1982:**

Herbert Simon (1982) was one of whom gave a good description for bounded rationality in economics, when he solved intractable problem by two different approaches. The first strategy when agents optimize by appropriate approximation for a given problem, and then he classified the optimum solution to the initial problem. The second strategy is to satisfy, i.e. the agent will check all possibilities of the solution in the first approach, until he finds the solution in which satisfactory is satisfied.

Simon describes two kinds of bounded rationality. The first one is optimizing the problem, and the second one is satisfying by finding the solution.

- **1993:**

Rabin gave us a very good reason for bounded rationality by building his fairness model between two players. Rabin developed his model by incorporating the beliefs, for two players with finite strategy game. It's called "psychological game".

- **2005:**

Ellison talked about the use of bounded rationality in the industrial organization, and he distinguished between three different types of bounded rationality in the industrial organization models. He applied rule – of – thumb approach in order to derive the optimal behavior as an optimal solution to the complex game that was constructed in the mind.

- **2009:**

I will continue my work on where Ellison ends, by constructing three rational models that are not mentioned in Ellison's article, and to explain the method that consumer adopts when he derives his optimal choice, and to know more about what do we mean by rule – of - thumb approach.

## Game Theory

Game theory is the study of multiperson decision problem. It is branch of applied mathematics that is used in the social sciences most notably economics. It was initially developed in economics to understand a large collection of economic behaviors, including behaviors of firms, markets, and consumers. In economics and philosophy, scholars have applied game theory to help in the understanding of good or proper behavior. The games studied in game theory are well-defined mathematical objects. Fudenberg and Tirole (1996).

Fudenberg and Tirole (1996) describe the game as follows: A game consists of a set of players, a set of moves (or strategies) available to those players, and a specification of payoffs for each combination of strategies. Most cooperative games are presented in the characteristic function form, while the extensive and the normal forms are used to define non cooperative games. Game theory attempts mathematically to capture behavior in strategic situations, in which an individual's success in making choices depends on the choices of others.

"Most economic applications of game theory use the concept of Nash Equilibrium or one of the more restrictive equilibrium refinements". (Fudenberg and Tirole 1996, 45). Traditional applications of game theory attempt to find equilibrium in these games-sets of strategies in which individuals are unlikely to change their behavior, most famously the Nash equilibrium. Game theory provides a theory of economic and strategic behavior when people interact directly, rather than "through the market."

In game theory, "games" have always been a metaphor for more serious interactions in human society.

In these serious interactions, as in games, the individual's choice is essentially a choice of a strategy, and the outcome of the interaction depends on the strategies chosen by each of the participants. On this interpretation, a study of games may indeed tell us something about serious interactions.

Game theory explained the human rationality by different games. Although the rational behavioral depends absolutely on the assumption of maximum utility for the players when players derive it, the players during some interesting games and because of their rationality may lead to loss more mounts of their material payoffs. However, if they cooperated in these games, or at least they concern about the other player strategies, they will be better off. Prisoner Dilemma is one of the good examples that rationality and self – interest will be worst off for both players.

The first known use is to inform us about how actual human populations behave. Some scholars believe that by finding the equilibrium of games they can predict how actual human populations will behave when confronted with situations analogous to the game being studied.

This particular view of game theory has come under recent criticism. First, it is criticized because the assumptions made by game theorists are often violated. Game theorists may assume players always act in a way to directly maximize their wins (the Homo Economics model), but in practice, human behavior often deviates from this model. Explanations of

this phenomenon are many: irrationality, new models of deliberation, or even different motives (like that of altruism).

Game theorists respond by comparing their assumptions to those used in physics. Thus while their assumptions do not always hold, they can treat game theory as a reasonable scientific ideal akin to the models used by physicists. However, additional criticism of this use of game theory has been levied because some experiments have demonstrated that individuals do not play equilibrium strategies. There is an ongoing debate regarding the importance of these experiments.

Alternatively, some authors claim that Nash equilibrium do not provide predictions for human populations, but rather provide an explanation for why populations that play Nash equilibrium remain in that state.

On the other hand, some scholars do not see game theory as a predictive tool for the behavior of human beings, but as a suggestion for how people ought to behave. Since Nash Equilibrium of a game constitutes one's best response to the actions of the other players, playing a strategy that is part of Nash equilibrium seems appropriate. However, this use for game theory has also come under criticism:

- First, in some cases it is appropriate to play a non-equilibrium strategy if one expects others to play non-equilibrium strategies as well.
- Second, In the Prisoner's Dilemma, each player pursuing his own self-interest leads both players to be worse off than had they not pursued their own self interests. Tucker's invention of the Prisoners' Dilemma example was equally important. (Smith)

## Prisoners' Dilemma

Tucker began with a little story, like this: two burglars, Bob and Ali, are captured near the scene of a burglary and are given the "third degree" separately by the police. Each has to choose whether or not to confess and implicate the other. If neither man confesses, then both will serve one year on a charge of carrying a concealed weapon. If each confesses and implicates the other, both will go to prison for 10 years. However, if one burglar confesses and implicates the other, and the other burglar does not confess, the one who has collaborated with the police will go free, while the other burglar will go to prison for 20 years on the maximum charge.

The strategies in this case are: confess or don't confess. The payoffs (penalties, actually) are the sentences served. We can express all this compactly in a "payoff table" of a kind that has become pretty standard in game theory. Here is the payoff table for the Prisoners' Dilemma game:

		Ali	
		confess	don't
Bob	confess	10 , 10	0 , 20
	don't	20 , 0	1 , 1

Each prisoner chooses one of the two strategies. In effect, Ali chooses a column and Bob chooses a row. The two numbers in each cell tell the outcomes for the two prisoners when the corresponding pair of strategies is chosen. The number to the left of the comma tells the payoff to the

person who chooses the rows (Bob) while the number to the right of the column tells the payoff to the person who chooses the columns (Ali).

Thus (reading down the first column) if they both confess, each gets 10 years, but if Ali confesses and Bob does not, Bob gets 20 and Ali goes free. So: how to solve this game? What strategies are "rational" if both men want to minimize the time they spend in jail? Ali might reason as follows: "Two things can happen: Bob can confess or Bob can keep quiet. Suppose Bob confesses. Then I get 20 years if I don't confess, 10 years if I do, so in that case it's best to confess. On the other hand, if Bob doesn't confess, and I don't either, I get a year; but in that case, if I confess I can go free. Either way, it's best if I confess. Therefore, I'll confess."

But Bob can and presumably will reason in the same way -- so that they both confess and go to prison for 10 years each. Yet, if they had acted "irrationally," and kept quiet, they each could have gotten off with one year each. In the Prisoners' Dilemma game, to confess is a dominant strategy, and when both prisoners confess, that is dominant strategy equilibrium.

The strategic behavior for both players will be different if the Prisoners' Dilemma game is repeated more than one time. Repeating the game will allow players to learn more about the game, and learning the game will encourage players to have new behaviors for playing the game, since they will know that the strategy "confess" will be dominated by "don't" in order to serve only one year in the prison instead of ten years.

## **Chapter Three: Literature Review**

- 3.1 Rationality (Herbert Simon 1955)
- 3.3 Incorporating Fairness into Game Theory and Economics
- 3.4 Fairness Model (Rabin 1993)
- 3.7 Bounded Rationality
- 3.8 Bounded Rationality in Industrial Organization  
(Glenn Ellison 2006)



## **Rationality**

Economic is related to game theory mainly by rationality. Neoclassical economics is based on the assumption that human beings are absolutely rational in their economic choices. Specifically, the assumption is that each person maximizes her or his rewards profits, incomes, or subjective benefits in the circumstances that she or he faces.

The concept of rationality has more than point of view for definition, for example in Herbert Simon (1955) literature "A Behavioral Model of Rational Choice" is to construct the definition of "rational choice". Simon (1955) in his article depends on the assumption of limited human capacity for computation in his motivation for bounded rationality.

He talks about the economic man; Simon describes the economic man by rationality. This economic man subtracts some notations and examines them in a rational way before taking decision. Simon suggests also if we can replace the rational man by another one without perfect rationality or kind of rational behavior, in order to bring him under the access of information and computational capacity. Simon defines the behavior by talking about the rational choice, he said that the rational choice depends on the following three terms:

1. The set of all alternative choices.
2. The relationship that determine payoff (satisfaction).
3. The preferences- ordering between payoffs.

Thus, selecting one choices and rejection the others includes that the selected one is done by applying rational organism – optimize choices and select the maximum one or minimum one according to the problem - and which choices must be fixed.

Simon indicated that the human behavior refer to that organism itself with its environment. The optimizing process for the problem may be psychological, and the organism has a limited computational capacity.

Simon built his rational model by identifying the following:

1.  $A$  : The set of all behavioral alternatives (choices or decision).
2.  $\overset{\circ}{A}$  is subset of  $A$ , called organism (considers).
3.  $S$  : The set of possible outcomes of the behavioral choices.

For simplicity,  $S$  is the set of ordered preference  $s_1 \prec s_2 \prec s_3 \prec \dots$

4. Payoff function  $V(s), s \in S$  that represents the value or the utility by the organism depending on each possible outcome of choices.
5. For any alternative chosen behavioral  $a \in A$  or  $a \in \overset{\circ}{A}$ .

Let  $S_a$  be the set of possible outcomes related to choose a.

6. For any element  $s \in S_a$ , define  $P_a(s)$  to be the probability that  $s$  will occur if  $a$  is chosen. Where  $P_a(s)$  is real, non negative with sum =  $\mathbf{1}$ .

Now using these elements, Simon defined the procedure of rational choice as follows:

\*. Max- Min rule: For any selection  $a \in A$  or  $a \in \overset{\circ}{A}$ , the worst payoff for this selection is as follows:

$$V(a) = \min V(s), \quad \forall s \in S_a$$

The best payoff for the same selection is:

$$V(a) = \max V(s), \quad \forall s \in S_a$$

\*. Probabilistic rule: is the maximum expected value of  $V(s)$ :

$$V(a) = \max \sum_{s \in S_a} V(s) P_a(s), \quad \forall a \in A$$

So the selected behavior is when the maximum payoff occurred:

$$\text{Max } V(S_a), \quad \forall a \in A$$

So, the above model summarizes the rationality of human behavioral. The model describes purely the human own interest regardless interests of the other agents. However, the next page is another point of view to define rationality or rational choice for a player.

Simon suggested "Simple payoff function" in order to observe the behavior processes in human, that lead to simplification in the calculations of making choice. He assumed the payoff value -  $V(s)$ - has to take either:

- $(+1, 0)$  : For (satisfactory, unsatisfactory). And the example here is by defining  $S$  to be the set of all possible prices for a house that an individual is selling. If the seller regard \$15,000 as an "acceptance" price, then any presented price over this amount will be "satisfactory". Any presented price less than this amount of money will be "unsatisfactory". If the presented price is equally to the above amount (\$15,000) then the seller is indifferent between selling and none selling the house.

If the seller has two offers of prices like \$16,000 and \$2300, then he will prefer to accept the larger one although both of them are very satisfactory prices for the house. Thus, the simple payoff function is inadequate presentation of the choice situation. So if there is a number of buyers offering to buy the house, then there exist a sequence of offers and may have to accept or reject the offer before receiving the other.

In general, the seller may receive a sequence of pairs or n-tuples of offers. And the seller may accept the highest coordinate of n-tuples before receiving another n-tuples. If  $\mathbf{S}$  is the set of all elements corresponding to the n-tuples of offers, then  $V(s)$  would be  $\mathbf{1}$  whenever the highest offer in n-tuples exceeds the "acceptance price". So if  $W(s)$  is the general payoff function, then  $V(s)$  will be the satisfactory approximation function to  $W(s)$ .

Thus, the rational decision process defines as follows:

1. Let  $\mathbf{S}'$  be subset of  $\mathbf{S}$  that denote to all possible of outcomes that has a satisfactory payoff  $(V(s')) = \mathbf{1}$  for all  $s'$  in  $\mathbf{S}'$ .
  2. Search in the set  $\mathbf{A}$ , for behavior alternatives elements that have possible outcomes in  $\mathbf{S}'$ , such that whenever  $a \in \mathbf{A}$ ,  $\mathbf{a}$  maps on a set  $S_a$  that contains in  $\mathbf{S}$ . If the behavior alternative can be found by this procedure, then a satisfactory outcome can be assured.
- $(+1, 0, -1)$  : For (win, draw, lose). And the example here is by defining  $\mathbf{S}$  - to be the set of all positions in chess game for 20<sup>th</sup> move of "White". Thus,  $(+1)$  position is one in which white possesses a strategy leading to win whatever Black does.  $(0)$  position is one in which white can enforce a draw but not to win. And  $(-1)$  position is one in which Black force to win. By considering that  $\mathbf{A}$  is the set of moves available to white for his 20<sup>th</sup> moves and  $\mathbf{S}$  be the set of all position that might be reached say by 30<sup>th</sup> moves. Assuming that  $\mathbf{S}'$  be a subset of  $\mathbf{S}$  that contains all "won" positions. White selects a move,  $\mathbf{a}$ , that maps on  $\mathbf{S}'$ .

Fudenberg and Tirole (1996) describe the definition of the rationality in the two player's case of games by:

"Rational player will use only those strategies that are best responses to some beliefs; he might have about the strategies of his opponents." Or "a player can not reasonably play a strategy that is not a best response to some beliefs about his opponents' strategies. Moreover, since the player knows his opponents' payoffs, and knows they are rational, he should not have arbitrary beliefs about their strategies. He should expect his opponents to use only strategies that are best response to some beliefs that they might have. And these opponents' beliefs, in turn, should also not be arbitrary, which leads to infinite regresses.

In case of two players, the infinite regress has the form: "I'm playing strategy  $\sigma_1$  because I think player 2 is playing strategy  $\sigma_2$ , which is a reasonable belief because I would play it if I were player 2, and I thought player 1 was using  $\sigma'_1$ , which is reasonable thing for player 2 to expect because  $\sigma'_1$  is a best response to  $\sigma'_2$ , ..." (Fudenberg and Tirole 1996, 49). This means that the player is more serious about the strategies that his opponent will choose. This happens because the player knows that his opponent is also rational player. The following page will identify rationality or rational man by another view (from Rubinstein view).

Rubinstein (1998) said that rationality of individuals is a result of some related factors like: property rights, money, and highly competitive markets. These circumstances oblige person to take into account his maximizing rewards. Another thing is that because he thinks that other individuals are all rational (Rubinstein 1998, 121). Rubinstein describes the rational man by: "rational decision maker is an agent who has to choose an alternative after a process of deliberation in which he answers three questions:

- What is feasible?
- What is desirable?
- What is the best alternative according to the notion of desirability, given the feasibility constraints?"

One can always explain the choice of an alternative, from a given set, as an outcome of a process of deliberation in which that outcome is indeed considered the best.

Rubinstein mentioned that the process of finding the feasible alternatives and the process of preferences are independent. This means that, if the decision maker found one alternative which is better than other alternative in any set that contains both alternatives, then he will rank them identically when encountering any other decision problem in which these two alternatives are available.

Formally, the most abstract model of choice refers to a decision maker who faces choices from sets of alternatives that are subsets of some “grand set”  $A$ . A choice problem  $\tilde{A}$ , is a subset of  $A$ ; the task of the decision maker is to single out one element of  $\tilde{A}$ .

Rubinstein concluded the scheme of the choice procedure employed by the rational decision maker is as follows:

The primitive of the procedure is a preference relation  $\succ$  over a set  $\tilde{A}$ . Given a choice problem  $\tilde{A} \subseteq A$ , choose an element  $x^* \in \tilde{A}$  that is  $\succ$  optimal (that is,  $x^* \succ x, \forall x \in \tilde{A}$ ). Thus, the decision maker has in mind a preference relation  $\succ$  over the set of alternatives  $\tilde{A}$ . Facing a problem  $\tilde{A}$ , the decision maker chooses an element in the set  $\tilde{A}$ ,  $C_{\succ}(\tilde{A})$  denoted by, satisfying that:

$$C_{\succ}(\tilde{A}) \succ x, \quad \forall x \in \tilde{A}$$

Sometimes we replace the preference relation with a utility function,  $u: A \rightarrow R$ , with the understanding that  $u(a) \geq u(a')$  which is equivalent to  $a \succ a'$ .



Rubinstein identifies some of the assumptions that are related to the rational man procedure, they are:

- Knowledge of the problem: The decision maker has a clear picture of the choice problem he faces: he is fully aware of the set of alternatives from which he has to choose (facing the problem  $\tilde{A}$ , the decision maker can choose any  $x \in \tilde{A}$ , and the chosen  $x^*$  cannot be less preferred than any other  $x \in \tilde{A}$ ). (The chosen  $x^*$  cannot be outside the set  $\tilde{A}$ ).
- Clear preferences: The decision maker has a complete ordering over the entire set of alternatives.
- Ability to optimize: The decision maker has the skill necessary to make whatever complicated calculations are needed to discover his optimal course of action. His ability to calculate is unlimited, and he does not make mistakes. (The simplicity of the formula  $\max_{a \in \tilde{A}} u(a)$  is misleading the operation may, of course, be very complex).
- Indifference to logically equivalent descriptions of alternatives and choice sets: The choice is invariant to logically equivalent changes of descriptions of alternatives. That is, replacing one alternative with another alternative that is logically equivalent does not affect

the choice. If the sets A and B are equal, then the choice from A is the same as the choice from B.

In neoclassical economic theory, to choose rationally is to maximize one's rewards. From one point of view, this is a problem in mathematics: choose the activity that maximizes rewards in given circumstances. Thus we may think of rational economic choices as the "solution" to a problem of mathematics. In game theory, the case is more complex, since the outcome depends not only on my own strategies and the "market conditions," but also directly on the strategies chosen by others.

This is why do we need to study bounded rationality; we need to have models that have the ability to reflect bounded behavior for player or consumer. It's not easy to construct such models, because there are many direct and indirect factors or variables related to industrial organization that affect the models.

In particular, some of the factors are indirect, which means that it is not easy to measure it, or even to notice it, and if you found like these factor, we need to add it to the model by correct way. One example is human behavioral. I think this is an important thing and in the same time there are no final models that describe bounded rationality without conditions. The following model is called fairness model, describes bounded rationality with some conditions.

## **Incorporating Fairness into Game Theory and Economics**

In Rabin literature (1993) "Incorporating Fairness into Game Theory and Economics", Rabin proposes a useful distinction between behavior based on altruism and behavior based on fairness. This difference depends on whether deviations from individually rational behavior are conditional on beliefs about other agents. "If somebody is being nice to you, fairness dictates that you be nice to him. If somebody is being mean to you, fairness allows - and vindictiveness dictates - that you be mean to him" (Rabin 1993, 1281). On the other hand, pure altruism would imply unconditionally nice behavior and reciprocal altruism would only imply conditionally nice behavior, not conditionally mean behavior.

Rabin depends on three games where Fairness Equilibrium (FE) can be clearly distinguished from Nash Equilibrium (NE): Prisoner's Dilemma (PD), Battle-of-the-Sexes (BOS), and Chicken. His approach is based on the assumption of complete information, in order to compare FE to NE, instead of little restrictive Rationalizable Equilibrium (RE) based on the assumption of incomplete information.

There is a famous assumption in most economic literatures, which is: people take care only in their own, their self - interests, and do not care about the other's own or interests. However, in game theory point of view, this behavior leads to worst payoff in some good examples of games.

Rabin identified the definition of fairness in his literature by:

"People like to help those who are helping them and to hurt those who are hurting them ". And " if somebody is being nice to you, fairness indicates that you be nice to him. If somebody is being mean to you, fairness allow- and vindictiveness indicates-that you be mean to him"(1281).

He called the outcomes of such these behaviors by "fairness equilibrium". He explained types of fairness equilibrium by *mutual max* – given others persons behavioral, each person maximizes the other payoffs - and *mutual min* when given others persons behavioral, each person minimizes the other payoffs.

He said that when we have large payoffs, then FE is the set of NE (Nash Equilibrium), but when payoff is small, then FE (Fairness Equilibrium) is the set of mutual max, and mutual min.

Rabin develops his framework for incorporation by three stylized facts:

1. People are willing to sacrifice their own to help people who are being kind. This is because the existence of the altruism and cooperation among people.
2. People are willing to sacrifice their own to punish people who are being unkind. Which means that it is not only to refuse helping others, but also to punish them. The good example here is "ultimatum game", this game consists of two persons (presenter and decider) whom need to divide amount of money (X) between each of them according to the following rule: the presenter offers

some division of  $X$  to the decider, if the decider say "yes", then each of them take money according to the proposal. If the decider say "no" each of them do not get money.

So in this case, the decider either to accept the offer or do not take any money, but the presenter knows this situation for the decider, his result of pure self – interest will always divide the money in order to get a lot, and he never offers more than a very little amount of money to the decider, and the decider should accept the offer. However the decider has the willing to punish the presenter because the later is unfair, so he always reject by saying 'no'.

3. The above (1) and (2) points have a high effect on the person's behavior, since they reduce the payoff of the person i.e. these motivation are very costly. Moreover, if the amount of money that keeps fairness is too high, then people will not have the willing to sacrifice by this amount of money. So the above two points are satisfied if we talk about small amount of money that people sacrifice by it.

To be sure about that, consider the "ultimatum game":

- Suppose (  $X = \$1$  ), then the decider will always reject the offer, if the presenter offers ( $\$0.9$  ,  $\$0.1$ ), however:
- Suppose ( $X = \$10$  million), then the decider will accept the offer, if the presenter offers ( $\$9$  million,  $\$1$  million).

As a result of Rabin models for fairness, the following results are held in his model:

1. Any NE (Nash Equilibrium) that is mutual – max or mutual - min is also FE (Fairness Equilibrium).
2. If the material payoffs are small, then roughly an outcome is FE iff its mutual-max or mutual-min.
3. If the material payoffs are large, then roughly outcomes FE iff it's NE.

The above three remarks describe the relation between Fairness Equilibrium (FE) and Nash Equilibrium (NE). The main point that has to effect the relation between the above two equilibriums is the amount of money that will be scarified in order to have Fairness Equilibrium, or it will be explored to have Nash Equilibrium.

## Fairness Model (Rabin 1993)

- Supposes that  $X$  is a positive number.
- Suppose also we have two persons, they would like to go to the same event together. But each prefers different event.

This is the game:

		Player 2	
		Opera	Boxing
Player 1	Opera	$2X, X$	$0, 0$
	Boxing	$0, 0$	$X, 2X$

Example 1: Battle of the Sexes

Payoff is a function that depends on the moves of the players. Player 1 prefer (opera, opera), while player 2 prefer (boxing, boxing). Suppose that both players take care about the payoff of each other. This means that, if player 1 will help player 2, then player 2 will help player 1, and on the other hand, if player 1 will hurt player 2, then player 2 will hurt player 1.

Now suppose that player 1 believes that:

- Player 2 is playing boxing, and
- Player 2 believes that player 1 is playing boxing.

Then, player 1 knows that player 2 chooses his strategy in order to help both players, so player 1 knows also that player 2 is not mean by selecting boxing ( because he can do opera).

In fact, (boxing, boxing) is Nash Equilibrium, to see the importance of fairness, supposing that player 1 believes that:

- Player 2 will play boxing, and
- Player 2 believes that player 1 is playing opera.

Then, player 1 concludes that player 2 plays opera in order to lower his own reward, and player 1 also concludes that player 2 wants to hurt him. So player 1 will feel hostility toward player 2, and player 1 will play opera in order to hurt player 2 as a response to later action.

If the hostility is strong enough, then player 1 will scarifies his own reward to hurt player 2 by playing opera instead of boxing. So in this case if both players have a strong bad reaction, then (opera, boxing) is equilibrium. So, player 1 payoff's does not depend on the action that is taken, but it depends on the beliefs of the player 2 motives.

Now, if we analyze the game by the conventional way, then according to the player 1, he prefers strictly to play opera rather than boxing. On the other hand, player 2 prefers strictly to play boxing rather than opera. Thus (opera, boxing) is equilibrium, no matter what is the chosen payoff. However, this contradicts with equilibrium if we incorporate the beliefs to the game.



Rabin developed his model by incorporating the beliefs, for two players with finite strategy game. Their mixed strategies are  $S_1, S_2$  for player 1, 2 respectively. They are derived from pure strategies  $A_1, A_2$ . The payoff function is defined as follows:

$$\pi_i : S_1 \times S_2 \rightarrow R$$

$\pi_1$  : Player's 1 payoff.

$\pi_2$  : Player's 2 payoff.

Rabin constructed "psychological game" by assuming that each player plays his strategy (by expecting his utility) subject to the following three factors:

1. His strategy.
2. His beliefs about the other player's strategy choice.
3. His beliefs about the other player's beliefs for his strategy.

For each above cases, Rabin identified the following:

- $a_1 \in S_1$  : The strategy that is chosen by player 1.
- $a_2 \in S_2$  : The strategy that is chosen by player 2.
- $b_1 \in S_1$  : Player 2 belief's about player 1's strategy choice.
- $b_2 \in S_2$  : Player 1 belief's about player 2's strategy choice.
- $c_1 \in S_1$  : Player 1 belief's about what the player 2's believes player 1 strategy is?

- $c_2 \in S_2$  : Player 2 beliefs about what the player 1 believes player 2 strategy is?

To incorporate fairness into model, Rabin define the "Kindness function"  $f_i(a_i, b_j)$  that measure how kind player i is being to player j. (Rabin assumed that both players have a shared notion of kindness and fairness)

Now, if player i believes that player j chooses strategy  $b_j$ , how kind is player i being by choosing  $a_i$  ?

Player i payoff is a pair  $(\pi_i(a_i, b_j), \pi_j(b_j, a_i))$  from the set of all payoff's that are feasible if player j chooses strategy  $b_j$ . i.e. this set is:

$$\pi(b_j) = \{(\pi_i(a, b_j), \pi_j(b_j, a)), a \in S_i\}$$

Let:  $\pi_j^h(b_j)$  : be the player j's highest payoff in  $\pi(b_j)$  among all pareto efficient points.

$\pi_j^l(b_j)$  : The player j's lowest payoff in  $\pi(b_j)$  among all Pareto efficient points.

$\pi_j^e(b_j) = [\pi_j^h(b_j) + \pi_j^l(b_j)]/2$ : is the equitable payoff for player j.

Finally let  $\pi_j^{\min}(b_j)$  be the worst possible payoff for the player j in the set  $\pi(b_j)$ .

Rabin defined the player's i kindness to the player j by:

$$f_i(a_i, b_j) = \frac{\pi_j(b_j, a_i) - \pi_j^e(b_j)}{\pi_j^h(b_j) - \pi_j^{\min}(b_j)}$$

If  $\pi_j^h(b_j) - \pi_j^{\min}(b_j) = 0$ ,

then  $f_i(a_i, b_j) = 0$ .

Notes:

- $f_i = 0$  iff player i is trying to give player j his equitable payoff.
- $f_i > 0$ , then player i gives player j more than his equitable payoff.

This can be happen only if the set of pareto frontier of

$\pi(b_j)$  is non singleton, otherwise  $\pi_j^e = \pi_j^h$

- $f_i < 0$ , then player i gives player j less than his equitable payoff.

This can be happen in two ways:

1. Player i, is grabbing more than his share on the pareto frontier of  $\pi(b_j)$ . **Or**
2. Player i, is choosing an inefficient point in  $\pi(b_j)$ .

Now, Rabin identified player's i beliefs about how kind player j is being to him by:

$$\tilde{f}_j(b_j, c_i) = \frac{\pi_i(c_i, b_j) - \pi_i^e(c_j)}{\pi_i^h(c_i) - \pi_i^{\min}(c_i)}$$

If  $\pi_i^h(c_i) - \pi_i^{\min}(c_i) = 0$

then  $\tilde{f}_j(b_j, c_i) = 0$

$f_i(a_i, b_j)$ , and  $\tilde{f}_j(b_j, c_i)$  are normally distributed so their values belong to the interval  $[-1, \frac{1}{2}]$ . These two kindness functions are used fully to determine the player's preferences (utilities).

Each player  $i$  choose his strategy  $a_i$  to maximize his expected utility

$$U_i(a_i, b_j, c_i) = \pi_i(a_i, b_j) + \tilde{f}_j(b_j, c_i) * [1 + f_i(a_i, b_j)]$$

- If player  $i$  believes that player  $j$  is serving him badly,

( $\tilde{f}_j(b_j, c_i) < 0$ ), then the player  $i$  utility  $U_i(a_i, b_j, c_i)$  will be less than his material payoff  $\pi_i(a_i, b_j)$ , player  $i$  wish to serve player  $j$  badly by selecting his strategy  $a_i$  such that  $f_i(a_i, b_j)$  is low or negative.

- If player  $i$  believes that player  $j$  is serving him kindly,

( $\tilde{f}_j(b_j, c_i) > 0$ ), then player  $i$  wish to serve player  $j$  kindly by selecting his strategy  $a_i$  such that  $f_i(a_i, b_j)$  is high or positive. Players will trade off their preferences for fairness against their material. However, since the utility function is bounded below and bounded above, the bigger material payoff for the player, the less his ability to concern about fairness. So the behavioral in this game is sensitive to the material payoff.

Rabin defines the Fairness Equilibrium (FE) as follows:

- \* The pair of strategies  $(a_1, a_2) \in (S_1 \times S_2)$  is fairness equilibrium, if for  $i \neq j \quad i = 1, 2$

1.  $a_i \in \max_{a \in S_i} U(a, b_j, c_i)$

$$2. c_i = b_i = a_i$$

To apply this definition for the above example 1:

If  $c_1 = b_1 = a_1 = \textit{opera}$  and  $c_2 = b_2 = a_2 = \textit{boxing}$ ,

then player 2 feels hostility and  $f_2 = -1$ . Thus:

- Player 1 utility from playing opera is zero (with  $f_1 = -1$ )
- Player 1 utility from playing boxing is  $X-1$  (with  $f_1 = 0$ ).

So if  $X < 1$ , then player 1 prefers opera to boxing giving these beliefs, while player 2 prefers boxing to opera. Therefore, (opera, boxing) is equilibrium, but in this case, both players are hostile toward each other, so no cooperation.

On the other hand, if both players are cooperated, then there is no hostility, so (opera, opera) and (boxing, boxing) are equilibrium for all values of  $X$ . The following game (Prisoner's Dilemma) explains the conditions that are necessary in order to say when we have Nash Equilibrium then its fairness equilibrium.

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	4X , 4X	0 , 6X
	Defect	6X , 0	X , X

Example 2: Prisoner's Dilemma

Fairness, in this game, may lead each player to help (or to sacrifice) the other. If it's knowledge for each player to play (cooperate, cooperate), then each player knows that the other player sacrifices his own material payoff to help him. Thus each will play "cooperate" to help the other. This is because the material payoff that is expected from defecting is not so large. Moreover if  $X$  becomes so small ( $X < \frac{1}{4}$ ), then (cooperate, cooperate) is fairness equilibrium.

However, for any value of  $X$ , the Nash Equilibrium (defect, defect) is also Fairness equilibrium. This is so, because playing (defect, defect) means that each player knows that other player is not willing to sacrifice  $X$  in order to give the other  $6X$ . So both players are hostile, and both of them have the same desire to hurt the other. In this game both strategies (defect, defect) and (cooperate, cooperate) are fairness equilibriums.

In general, Rabin concluded that every Nash Equilibrium which is mutual - max or mutual - min is Fairness Equilibrium.

His proof was if  $(a_1, a_2)$  is NE, then both players are maximizing their material payoffs.

- First, if  $(a_1, a_2)$  mutual – max outcome, then  $f_1$  and  $f_2$  must be nonnegative. Thus, both payers have positive regard for the other. Since each player chooses his strategy in order to:
  1. Maximize his material payoff and
  2. Maximize the material payoff of the other player.

So, this maximizes the overall utility.

- Second, if  $(a_1, a_2)$  mutual – min outcome, then  $f_1$  and  $f_2$  must be non positive. Each player wants to decrease the amount of the material payoff of the other, in the same time, he play to maximize his own material payoff, so this must maximize his utility. The following page contains another example of games (Prisoner's Non – Dilemma). In which adding fairness keeps Nash equilibrium.



		<b>Player 2</b>
		<b>Cooperate</b>
<b>Player 1</b>	<b>Cooperate</b>	<b>4X , 4X</b>
	<b>Defect</b>	<b>6X , 0</b>

Example 3: Prisoner's Non - Dilemma

People determine the fairness of the other by their motives, not only by selecting their action. Motives can be inferred from player's choices among his available choices of strategies. (What he actually chooses differ from what he should have chosen)

The above example, player 2 is forced to cooperate and player 1 is always defecting. So the unique fairness equilibrium is (defect, cooperate). This contradicts with the equilibrium (cooperate, cooperate) in example 2. The reason for that is player 1 will not feel now for a positive regard to player 2 decisions, and player 2 does not present any favor for player 1.

In example 1 and 2, we saw that adding fairness to the game create new equilibrium, but without getting rid from Nash Equilibrium. However, the following example (Chicken Game) illustrated that adding fairness will get rid of Nash Equilibrium.

		Player 2	
		Dare	Chicken
Player 1	Dare	-2X , -2X	2X , 0
	Chicken	0 , 2X	X , X

Example 4: Chicken

Each country hopes to dare, while the other country hopes to break down. But both dread of the outcome (dare, dare). Note that (dare, chicken) and (chicken, dare) are both Nash Equilibriums.

Consider the Nash Equilibrium (dare, chicken), where player 1 plays dare and player 2 plays chicken. The question is: Is it fairness equilibrium?

Its common knowledge when player 1 is hurting player 2 by playing dare to help himself, so for small amount of X, neither (dare, chicken) nor (chicken, dare) is fairness equilibrium. In this case (small X) both Nash equilibriums are inconsistent with fairness. In small amount of X, both (dare, dare) and (chicken, chicken) are fairness equilibriums. So (dare, dare) is mutual – min while (chicken, chicken) is mutual – max, but neither of them is Nash Equilibrium.

For games with small material payoffs, finding Fairness Equilibrium is to approximate Nash Equilibrium verbalizing the following two points:

1. Each player wants to maximize the other material payoff.
2. Each player wants to minimize the other material payoff.

However, Games with large enough material payoffs, the player's behavioral is dominated by material self – interest. In particular, players will only play Nash Equilibrium as payoff become large.

This means that as payoff becomes larger, the player, in this case, will take this high material payoff into consideration, and the player can not ignore this amount of payoff. Thus, ignoring this high payoff will be considered as loss for the player and will not be considered as fairness.

## **Bounded Rationality**

Why do we need to study bounded rationality? What are the motivations for studying bounded rationality? The answer is due to the following points:

- 1- The first one is due to the assumption of perfect rationality. Making decisions is to maximize the utility "self-interest" regardless the others actions. From game theory point of view, consumer must avoid the assumption of fully rational in several games (by the interactions between players) in order to get their favorite payoffs. For example: In the Prisoner's Dilemma, if each player pursuing his own self-interest then this leads both players to be worse off rather than if they did not pursue their own self interests.
  
- 2- The second reason to study bounded rationality is the infeasibility of computation of perfect rationality. For example in chess game, both players can not draw their plans to achieve maximum expected payoff (utility) due to difficulties of calculations in one direction, and the other direction is due to limited human capacity for calculation. However, he can choose first favorite strategy, but he can not plan in his mind more than one or two best strategies.
  
- 3- Humans rarely exemplify the perfect rational model.  
The example here is the lotteries game.

The concept of bounded rationality was taken into consideration, because the assumption of perfect rationality faces some violation in some good examples of games. Perfect rationality does not give a reasonable result or it does not describe exactly the human rational behavioral during these games. However, thinking to do bounded rational models is still open, because there are some topics in industrial organization that can not bend for bounded rational models like human behavioral.

To show that people do not always follow the perfect rational behavior in order to maximize their utilities, we look at the result of psychological experiment, found in example of (Rubinstein 1998), in which choices between two lotteries are not full rational.

Suppose that we have four lotteries  $L_1, L_2, L_3, L_4$ , A lottery where  $x$  is awarded with probability  $P$ , and  $0$  is awarded with probability  $1-P$  is represented by  $(x, P)$ .

If we have to choose between the following two lotteries:

$L_1 = (4000, 0.2)$  And  $L_2 = (3000, 0.25)$ , then

$L_1$  Reward =  $0.2 * 4000 = 800$ , while

$L_2$  Reward =  $0.25 * 3000 = 750$ .

So the popular will choose  $L_1$ .

However, if we have to choose between the following two lotteries:

$$L_3 = (4000, 0.8) \quad \text{And} \quad L_4 = (3000, 1.0), \text{ then}$$

$$L_3 \text{ Reward} = 0.8 * 4000 = 3200, \text{ while}$$

$$L_4 \text{ Reward} = 1.0 * 3000 = 3000.$$

But the most common choice is  $L_4$ .

Now, choosing the following lotteries  $L_1$  and  $L_4$  contradict with the von Neumann- Morgenstern independence axiom. This axiom indicates that to choose between two lotteries with similar type of vectors, then people try to cancel the similar parts of two vectors. So when people compare between  $L_1$  and  $L_2$ , they see that 0.2 and 0.25 are closed to each other, so these two probabilities are cancelled, and people in this case look at the first component of each lotteries, and since  $4000 > 3000$  so people will choose  $L_1$ .

However, the comparison between  $L_3$  and  $L_4$  from people point of view make some sense, people will cancel the second parts of each components, and since  $4000 > 3000$ , but they will choose  $L_4$ . Yes there is risk aversion here, but if we calculated as reward using preferences then  $3200 > 3000$ , but people do not choose the lottery that gives them more reward. Thus, human behavior is not perfect rational in this example.

## **Bounded Rationality in Industrial Organization:**

The developments in this field of study started in the beginning of 1980s. Herbert Simon (1982), indicated to the kinds of bounded rationality, they are two, the first one is optimizing an approximation problem, and the second is satisfying or finding satisfactory solution to the problem.

Herbert Simon was one of whom given a good description for bounded rationality in economics, when he solves intractable problem by two different approaches. The first strategy is that agents optimize by appropriate approximation for a given problem, and then he classifies the optimum to be solution to the initial problem. (Simon 1982)

The second strategy is to satisfy, i.e. the agent will check all possibilities of the solution in the first approach, until he find the solution in which satisfactory are satisfied. Like a student who searches for a course project topic until he finds one that is good enough. However searching all possible topics will present the student to do his project. Simple payoff function that Simon suggested - that we describe before - is a good description for bounded rationality, but it stills has a problem when we have two or more satisfactory solution to the problem.

Simon believed that agents face uncertainty about the future and costs in acquiring information in the present. These factors limit the extent to which agents can make a fully rational decision, thus they possess only “bounded rationality” and must make decisions by “satisfying,” or choosing that which might not be optimal but which will make them happy enough.

However, Ellison (2006) has comments on Herbert Simon literature "A Behavioral Model of Rational Choice". Ellison said that "The progress he makes toward this goal is less satisfying. He mostly ends up emphasizing a “satisfying” model in which agents search for actions until they find one that achieves a payoff that provides them with at least some aspiration level".

Ellison saw this rule – of - thumb approach when playing equilibrium of a game is as follows:

- Fully rational approach: in which players learn how to play equilibrium of a game.
- Boundedly rational approach: in which rules lead or not lead to play equilibrium.

Another good example that used industrial organization stories is Mobius (2001) who explains the effect of changing competition in U.S.A over the local telephone services using a rule – of – thumb model. In this model there is a connection between consumers and business by social networks. He divided the networks into several regions each contains consumers and one business. The behavioral rule tells us how consumers and business decide within competition.

In case of consumers, they take their choices (ether to buy service or no phone) when other actions are given. Firms have similar rules. While business choose second phone in order to connect consumers in other regions.



Recent research usually attributes the irrational behavior to consumers and not to firms. The reason is that a firm is more careful in its decision making, several people are often involved in making important decisions, and the firm can hire consultants to help it overcome its irrationality if such irrationality exists.

Moreover, irrationality on the side of the firm should reduce its profits, hurt the firm's position both in its product markets and in the capital markets in which it obtains its financing, and eventually is likely to lead to the firm's bankruptcy due to more rational firms driving it out of business.

Therefore it is hard to believe that firms can behave significantly in an irrational fashion and still survive the competition for a long time. It is much more plausible that consumers behave irrationally, since the above considerations do not apply to them. Consumers do not disappear if they make biased decisions; they just do not obtain a utility level as high as they could get with optimal decision making.

A new strand in the industrial organization literature, behavioral industrial organization, addresses ways to depart from the assumption of fully rational, utility maximizing agents. A nice overview of this new approach and its predecessors is Ellison (2005) who mentioned that we can do models like rational models but using the assumption of favorite for consumers as behavioral biases instead of the assumption of maximum utility.

Moreover Ellison (2006) talks about irrationality, he joints irrationality to consumer, and he describes the firm's exploration when consumers are irrational.

Ellison (2006) literature "Bounded Rationality in Industrial Organization" talks about the use of bounded rationality in industrial organization. In fact, the literature on bounded rationality in IO is so sparse, so IO economists are so heavily invested in the rational game – theoretic approach in order to make it complete field of study. Ellison distinguishes between three different traditional types of bounded rationality in IO models:

The first one is called rule – of – thumb approach (Agent who enter in simple rules-of - thumb to make choices). In this approach he assumes that consumers or firms behave easily to maximize the profit by following rule – of – thumb, so it's used to model irrationality by deriving behavior to maximize satisfaction. For example when a consumer wants to buy something from a super market, he asks himself several questions about this product; thus, the consumer has a very complex game in his mind. However, the rational decision by consumer is easily obtained to optimize his utility.

Rule – of – thumb approach is similar to game theory approach, "in which one posits utility functions and derive the behavior as optimizing choices." (Ellison 2006, 6) In this case, rule – of – thumb approach has two advantages as Ellison mentioned: it's unbelievable that agents do rational calculations and it solves easily a complex game.

Rule – of – thumb approach assume that consumer do rational calculations, and the rational behavior will be solution to the game, which contradict mathematics in game theory point of view. This is first short cut of rule – of – thumb approach, and the second one is when game

theory attested developments through industrial organization, "rule – of – thumb literature could not withstand the onslaught." (7)

The second one is called Explicit bounded rationality, (Agent who find it costly to make decisions) Ellison in this literature indicated that "cognition is costly, and agents adopt second-best behaviors taking these costs into account." In which consumers assumed to satisfy instead of maximize.

The last tradition that Ellison discussed is the effect of behavioral biases in consumers from psychology and economic point of view on the setting of industrial organization (Agent who exhibit behavioral biases).

In this context Ellison mentioned how Joskous (1973) estimate the profit model as an example of behavioral to industrial organization.

$Y_i = X_i B + \epsilon_i$ , where  $Y_i$  is the electric rate in New York Public Service Commission,  $X_i$  firms earning growth, B is the parameters and  $\epsilon_i$  are normally distributed.

The probit model has the ability to estimate firm's profit.

$\pi_i = X_i B + \epsilon_i$ , where  $\epsilon_i$  are unobserved profit.

- If  $\pi_i$  is linear, then Joskous assume that when  $\pi_i > 0$ , firms apply for a rate hike.
- If  $\pi_i$  is not linear: Ellison said that "firms follow an irrational rule – of – thumb" to apply for a rate hike when  $\pi_i > 0$ .

In this case regression model dose not estimate profit, but it still estimate the behavior.

Boundedly rational approach has two advantages in Ellison point of view, first one is that behavioral of boundedly rational is tangible in several games, and the other is the "tractability" of boundedly rational models has the ability to enable the process of incorporating other variables in the model." Incorporating the social networks" in Mobius model (2001).

Ellison in his article presents an important matrix that consists of rows that contains "standard IO models" and columns that contains "different biases ". The matrix will be taught in high –level –courses over the next few years, and to fill this matrix is to build new models in IO.

However the result matrix (between industrial organization models and the behavioral biases), if it was done, may be it will help consumers to avoid the fully rational assumption. However, if there were such biases behaviors, then firm's decisions for their prices and products will take place to explore these biases if there are no competition between firms in order to delete this exploration.

Ellison indicated that if consumers have  $\epsilon$  additional cost to depart from the assumption of fully rational, then prices will scatter in all dimensions from the competitive level to the monopoly level. Diamond (1971) and Ellison (2002) are good examples that explain the effective impact on prices due to small changes on the biases of consumer behavior. On the other hand, this phenomena stay without sensitivity in other models, means that small change in the behavior biases will lead to small change on the prices.

## **Chapter Four: Generalized Models for Bounded Rationality**

4.1 Introduction

4.2 Rational Model (I)

4.3 Rational Model (II)

4.4 Rational Model (III)

4.5 Bounded Rationality in Rational Models

## **Introduction**

The following chapter contains my work. I will start my work from where Ellison ends. Here we have three rational models that describe how the optimal choice will be derived among three different models.

When we entered a super market and we saw new product that we have never seen before we will apply a rule – of – thumb approach according to Ellison literature (2006), in order to derive our optimal behavior as an optimal choice. Ellison in his approach assumes that consumers behave easily to solve the complex game in their minds in order to get their optimal behaviors, the question now: What kind of games that consumers construct in their minds? And how the problem is solved easily? The answers of these questions are not mentioned in Ellison's literature, so the first model will solve the problem that is constructed in our mind by describing the cases where the decision takes place.

The second two models will describe how bounded rationality can be applied in firms by different procedures. For example in the second model we have only one monopoly firm, so how firm - in this case – can behave in order to encourage customers to buy the product which have small selling share. The firm here may enforce customers to do bounded rationality using the full rational approach.

In the last model firms within competition can do bounded rationality by scarifying small amount of their profits in order to attract people to their product, and to keep their effectives in the market.

## Rational Model (I)

Assumptions:

1. Suppose that we have only one player.
2. Assume the player saw only one new product when he entered a supermarket.
3. The player has never seen the new product before.

According to Ellison, the player will apply a rule – of – thumb approach in order to derive his behavior as an optimal choice to buy or not to buy this new product.

The Question is how to derive the rational choice?

Now: when the player applies a rule – of – thumb approach, he will ask himself the following questions about this new product, say:

$$X_1, X_2, \dots, X_n$$

Where:

$X_1$  : The color of the new product.

$X_2$  : The price of the new product.

$X_3$  : The shape of the new product.

$X_4$  : The player Income.

$X_5$  : The advice of one friend about the new product.

$X_6$  : The player needs to buy other products.

$X_7$  : The validity of the new product.

$X_8$  : The extent of benefiting from the new product.

$X_9$  : The player hasting.

$X_{10}$  : The method of showing new product.

.

.

.

$X_n$

These are the player questions that he begins to ask himself when he is looking to the new product.

In general these X's have different effects on the player decision, so we need to define weight variables ( $W_i$ ) for each

$X_i$ ,  $i = 1, 2, 3 \dots n$ . Thus, in this case, we give each variable suitable weight in order to represent its real effect on the player decision.

In order to study how we derive the optimal behavior (decision of buying or not to buy the new product) for the rational player, we did a separation for X's like that:



For each  $X_i, i = 1, 2, 3 \dots n$ .

$X_i$  Has a positive effect on the player decision with probability  $P_i$

$X_i$  Has a negative effect on the player decision with probability  $1-P_i$

Where  $P_i \in [0, 1]$ .

Now suppose that:

$\vec{X}_P : X_1, X_2, \dots, X_k$  Have positive effect on the player decision. And

$\vec{X}_N : X_{k+1}, X_{k+2}, \dots, X_n$  Have negative effect on the player decision.

So the player's decision is positively related to the effect of  $X_1, X_2, \dots, X_k$ , while player decision is negatively related to the effect of  $X_{k+1}, X_{k+2}, \dots, X_n$ .

In fact, if we can do models to study how players derive the optimal behaviors, then bounded rationality can be easily described when the player cancel one of these X's or decrease the weights for some X's. In this context, firms exploration to consumers biases in their behavior depends on which X's the consumer will cancel, or which X's that the consumer minimize their weights.

Now: let A be the set of all alternative X's.

Define the "Utility Function":

$$U : A \rightarrow \mathbb{R}$$

So that U is "separable", thus, we have three cases:

Case 1: if  $U(\vec{X}_P, 0) > U(\vec{X}_N, 0),$

Then the player will decide to buy.

Case 2: if  $U(\vec{X}_P, 0) < U(\vec{X}_N, 0),$

Then the player will not buy.

Case 3: if  $U(\vec{X}_P, 0) = U(\vec{X}_N, 0),$

Then the player has "Purely random choice"  
between the purchases the product or not.

This case tell us the player is naïve, he dose not have the ability to decide, and he does not know his true preference

i.e. He will buy with 50% probability.

He will not buy with 50% probability.

The utility of the alternative X's will be:

$$U ( X_i ) = \begin{cases} W_i P_i \dots \dots \dots \text{for the..} \text{"psitive..effect"} \dots \dots i = 1, 2, \dots k \\ W_i (1 - P_i) \dots \dots \dots \text{for the..} \text{"negative..effect"} \dots \dots i = k + 1, k + 2, \dots, n \end{cases}$$

So;

\* The utility of positive effect:

$$\begin{aligned} U ( X_1, X_2, \dots, X_k ) &= \sum_{i=1}^k U ( X_i ) \\ &= \sum_{i=1}^k W_i P_i \end{aligned}$$

\* The utility of negative effect (Disutility):

$$\begin{aligned} U ( X_{k+1}, X_{k+2}, \dots, X_n ) &= \sum_{i=k+1}^n U ( X_i ) \\ &= \sum_{i=k+1}^n W_i (1 - P_i) \end{aligned}$$

Now if we look at the three previous cases, we have:

Case 1: the player will decide to buy if:

$$U ( X_1, X_2, \dots, X_k ) > U ( X_{k+1}, X_{k+2}, \dots, X_n )$$

$$\sum_{i=1}^k U ( X_i ) > \sum_{i=k+1}^n U ( X_i )$$

$$\sum_{i=1}^k W_i P_i > \sum_{i=k+1}^n W_i (1 - P_i)$$

$$\sum_{i=1}^k W_i P_i > \sum_{i=k+1}^n W_i - \sum_{i=k+1}^n W_i P_i$$

$$\sum_{i=1}^k W_i P_i + \sum_{i=k+1}^n W_i P_i > \sum_{i=k+1}^n W_i$$

$$\sum_{i=1}^n W_i P_i > \sum_{i=k+1}^n W_i$$

This means that: the player will decide to buy if:

Sum of positively weight  $>$  Weight sum of negative effects

i.e., before the player make his decision; he will do some calculation in his mind as follows: he will sum the positive effect of all factors according to their probability as above, and compare the result with the sum of the weights for only those factors that have negative effect on his decision. If the later one is less than the first one, he will decide to buy.

Case 2: The player will not buy if:

$$\sum_{i=1}^n W_i P_i < \sum_{i=k+1}^n W_i$$

Case 3: the player will choose randomly if:

$$\sum_{i=1}^n W_i P_i = \sum_{i=k+1}^n W_i$$

Although we think it's really what happens in the player mind before he is going to decide to buy, we are very strange from the player calculations, because the player will decided to buy or not to buy quickly.

In fact, when the player see new thing that needs acceptance to "buy" or to say "yes", he puts in his mind a standard level of satisfaction or utility that are related to that thing, and he begins to calculate its benefit in his mind as above by finding the positive effect, so if the positive effect is greater than his level of satisfaction, then he decides to buy or he will say "yes", otherwise he will not buy its product and he will say "no" if his problem.

## Rational Model (II)

The above explanation is when we have only one new product and only one buyer. The question now is: what happen or how the human behavior is derived, if we have more than one new product?

The following page will explain the above issues by identifying the set of new cars, and assuming that we have also one buyer, the buyer behavioral will be derived by comparing the utility of related factors according to the buyer satisfaction (utility).

Suppose that  $A$  is the set of all preferred products  
(Say  $A$  is a set of  $(m)$  new different cars).

- Suppose that we have one buyer.
- We have only one monopoly firm.
- Suppose he wants to buy a new car, so how the buyer chooses his suitable or favorite car?

For any chosen product  $i$  ( $car_i$ ) in  $A$ , we will build the following table in order to explain the derivation of real human behavioral as a rational behavior to achieve his best expected utility by maximizing the material payoff.

The product	Buyer Calculations (Approximations)					Seller Price	Difference
	Motivations Or Shortcomings	Reward (Value)	Probabilities	Costs	Total (Costs)	Actual Price	(Payoff/Loss)
Car 1	S11	R11	P11	R11P11	$C_1$	$P_1$	$\mathbb{J}_1 = C_1 - P_1$
	S12	R12	P12	R12P12			
	S13	R13	P13	R13P13			
	⋮	⋮	⋮	⋮			
	S1n	R1n	P1n	R1nP1n			
Car 2	S21	R21	P21	R21P21	$C_2$	$P_2$	$\mathbb{J}_2 = C_2 - P_2$
	S22	R22	P22	R22P22			
	S23	R23	P23	R23P23			
	⋮	⋮	⋮	⋮			
	S2n	R2n	P2n	R2nP2n			
⋮	⋮	⋮	⋮	⋮	⋮	⋮	
Car m	Sm1	Rm1	Pm1	Rm1Pm1	$C_m$	$P_m$	$\mathbb{J}_m = C_m - P_m$
	Sm2	Rm2	Pm2	Rm2Pm2			
	Sm3	Rm3	Pm3	Rm3Pm3			
	⋮	⋮	⋮	⋮			
	Smn	Rmn	Pmn	RmnPmn			

Where:

\*  $S_{ij}$  is the buyer strategies for  $car_i$  (when he chooses the product  $i$ ),  
 $j = 1, 2, 3 \dots n$

For example: if we look at the first car ( $i = 1$ ), then we suppose that:

$S_{11}$ : The color of the car1.

$S_{12}$ : The speed of the car1.

$S_{13}$ : The shape of the car1.

·  
·  
·

$S_{1n}$

\*  $R_{ij}$  is the reward (value) in which buyer identifies when he  
chooses  $S_{ij}$ . (The value (cost) of  $S_{ij}$ ) its like weights.

\*  $P_{ij}$  is the probability of buying the  $car_i$  by the buyer because of  
the effect  $S_{ij}$  when he chooses  $car_i$ .

Note that:

$$\sum_{j=1}^n P_{ij} = 1$$

For every  $i = 1, 2, 3 \dots m$



\* The Total cost for each car is as follows:

$$C_i = \sum_{j=1}^n R_{ij} P_{ij} \quad i = 1, 2, 3, \dots, m$$

This cost  $C_i$  means that I have the ability to accept buying car i with at most this cost, otherwise I will not buy this car.

- The buyer starts his bargaining by looking at the cars, one by one, and he begins his mechanism for calculations as in the above table, in order to find the global cost for each car alone.
- Maximum payoff (positive difference) along all cars ( $i = 1, 2, 3 \dots m$ ), will determine the optimal behavioral for the buyer. (Which new car he prefers to select?).

This method of derivation is done by a full rational choice by the buyer.

The buyer is evaluating now. He looks only to those cars that have a positive difference (payoffs) as in the above table. And then he finds the maximum positive difference (payoff) as follows:

$$\text{Max } \pi_i = \max [C_i - P_i], \quad i = 1, 2, 3, \dots, m$$

Suppose that he found his maximum value at  $i = k$ , so he buys car (k). This means that after his approximations value for this car he concludes that this car worth's only  $\pi_k$ , but the seller sells the car at  $P_k$ , and since

the difference  $\pi_k - P_k$  is largest positive value among all positive differences, then the buyer will gain this difference (payoff) after buying the car.

If the last column in above the table (difference) is negative for all cars, then the buyer will not buy any car. Because in this case, the last column represents a loss expectation for the buyer.

Last, if the buyer calculates the maximum value of this difference, and he found its zero, and it's satisfied at say  $car_t$ , then this means that his approximation cost to this car is closed to the seller price. In this case, he will buy the car without gaining or losing any difference.

Suppose by chance that the maximum material of payoff of buyer approximations is satisfied at more than one car say:

$$car_1, car_2, \dots, car_s \text{ Where } s \leq m$$

Then, the buyer will decide which car from these cars he wants to buy. Since all of them have the same payoff according to buyer, then each of them has the same opportunity - the same probability- that is  $(\frac{1}{s})$ . The buyer, in this case, is purely random choice to choose between these cars.

All of the above calculation under the assumption of existing one monopoly firm that is producing (or selling) cars. The firm will notice the kind of cars that buyers prefer to buy. But firms need to sell the other

kinds of cars that usually have less preference from buyers than others. So what firm should do in this case?

We can conclude some procedures from the above table that the firm can do in order to improve the process of selling other kinds of cars. These procedures depend on the buyer calculations (approximations) in one way, and they depend on the firm pricing itself.

Since we have a monopoly firm- no competition here -according to the firm procedures, the firm can easily increase the prices of the preferred cars. But this behavior has positive/negative effect on the firm:

- Positive: if the last column in the above table still has a positive payoffs (or non negative payoffs), then buyer will buy the same car or he will change his opinion to another kind of cars according to his maximum material payoff.
- Negative: if the last column in the above table become negative (has no positive payoffs), then buyer will not buy any car, and the firm will not have selling cars.

So increasing the prices of preferences cars may hurt the firm and the buyer. And this will not affect so much the other kinds of cars.

Suppose the firm decreases the price of all cars by small amount, it's clear from the above table that there will be more buyers who have positive maximum payoffs; the firm will sell more cars from different types.

If the firm has the willing to sell the other kinds of cars (which have less preferences), firm should look at the buyer strategies as in the above table. The firm should improve these strategies by adding some services, prizes, advertisements or motivations to those kinds of cars in order to encourage buyer to buy them.

Indeed, the buyer will look for these cars by another view, he will increase the costs of his approximations to these cars, so the buyer in this case, will have a larger costs (C), this gives the firms more opportunity that the buyers will increase their ability to buy these cars.

## Rational Model (III)

Now, suppose that we have more than one firm – here the competition exists – the firm policy will be different. But the buyer will do his calculations (approximations) by the same way by adding some new strategies to the table that are related to the facilities or difficulties that firm present to buyers. The firm will differ in their policies to attract buyers. And price discriminations may exist between same cars in different firms.

The following table summarizes the buyer behavior among all firms. He will do the same calculations for each car in all firms, in order to maximize his payoff (difference), and then he will compare his payoff for same car among all firms.

Now, if the buyer likes to buy a special car, for example car ( $i$ ),  $i \leq m$ , and there is more than one firm for selling cars, then what he easily like to do is just to look at the row  $i$  in the below table and he buys his special car from the firm that exist in the last entry in the row  $i$ , i.e., he will buy his car from the firm that gives him the highest maximum payoff among all maximum payoff of the other firms.

**For different firms, the buyer will do the following:**

		Firms					
Cars		Firm 1	Firm 2	Firm 3	...	Firm k	Max Row
Buyer	Car 1	<p style="text-align: center;">Max <math>J_{11}</math></p> <p>Max payoff if the buyer wants to buy the car1 from firm 1</p>	<p style="text-align: center;">Max <math>J_{12}</math></p> <p>Max payoff if the buyer wants to buy the car1 from firm 2</p>	<p style="text-align: center;">Max <math>J_{13}</math></p> <p>Max payoff if the buyer want to buy the car1 from firm 3</p>	...	<p style="text-align: center;">Max <math>J_{1K}</math></p> <p>Max payoff if the buyer wants to buy the car1 from firm k</p>	The firm that has Max – payoff For car 1 Among All firms According to The buyer
	Car 2	<p style="text-align: center;">Max <math>J_{21}</math></p> <p>Max payoff if the buyer wants to buy the car2 from firm 1</p>	<p style="text-align: center;">Max <math>J_{22}</math></p> <p>Max payoff if the buyer wants to buy the car2 from firm 2</p>	<p style="text-align: center;">Max <math>J_{23}</math></p> <p>Max payoff if the buyer wants to buy the car2 from firm 3</p>	...	<p style="text-align: center;">Max <math>J_{2K}</math></p> <p>Max payoff if the buyer wants to buy the car2 from firm k</p>	The firm that has Max – payoff For car 2 Among All firms According to The buyer
	Car 3	<p style="text-align: center;">Max <math>J_{31}</math></p> <p>Max payoff if the buyer wants to buy the car3 from firm 1</p>	<p style="text-align: center;">Max <math>J_{32}</math></p> <p>Max payoff if the buyer wants to buy the car3 from firm 2</p>	<p style="text-align: center;">Max <math>J_{33}</math></p> <p>Max payoff if the buyer wants to buy the car3 from firm 3</p>	...	<p style="text-align: center;">Max <math>J_{3K}</math></p> <p>Max payoff if the buyer wants to buy the car3 from firm k</p>	The firm that has Max – payoff For car 3 Among All firms According to The buyer
	⋮	⋮	⋮	⋮	⋮	⋮	⋮
	Car m	<p style="text-align: center;">Max <math>J_{m1}</math></p> <p>Max payoff if the buyer wants to buy the car m from firm 1</p>	<p style="text-align: center;">Max <math>J_{m2}</math></p> <p>Max payoff if the buyer wants to buy the car m from firm 2</p>	<p style="text-align: center;">Max <math>J_{m3}</math></p> <p>Max payoff if the buyer wants to buy the car m from firm 3</p>	...	<p style="text-align: center;">Max <math>J_{mk}</math></p> <p>Max payoff if the buyer wants to buy the car m from firm k</p>	The firm that has Max – payoff For car m Among All firms According to The buyer

On the other hand, if the buyer does not matter any special car in his mind, and there is more than one firm in the market, then the buyer will do maximization for his expected utility by maximizing the last column in the above table. Thus, he will find the suitable firm in which the payoff for the favorite car is closed to his maximization.

In this case, the buyer will do the following: Max (max row) to get or to choose the firm that gives him maximum payoff among all firms. In fact, this is what happens in real life. The buyer will move from firm to another in order to find the best firm- the firm that gives the buyer his request with minimum cost according to the buyer- that he prefers in one case, and in another case to a achieve the best response from this firm.

However, the buyer may have some error in his calculations. If the buyer has few biases in his optimal response (choice) for himself, and if such biases exist, then we explain this by human irrationality. Biases are related to which level of calculations the buyer is biased. These levels of approximation are consisting of his strategies, reward (values), probabilities and his cost approximation that are appeared in the above table.

These biases that buyer may face in his approximation have the big influence on the decision of buyer for which product (car) he would be able to buy within competition of firms. For example, if the buyer biases lead to increase his cost approximation in any firm, then the buyer total cost will increase, and this will decrease his material payoff in that firm. Thus, such biases may force the buyer to change the selling firm. On the other hand, if such biases lead to decrease the total cost, then this will increase the total payoff. The same thing, this will easily change the selling firm according to the buyer who will buy from that firm.

## **Bounded Rationality in Rational Models**

As we saw in the previous three models of rationality, the buyer did his calculations or approximations in his mind in order to maximize his expected utility by full rational way.

First of all, I will identify some important issues that help the human to derive from perfect rational behavioral. The first issue is the future economic news about the prices of the preference product. To see the effects of such issue on the human behavioral; suppose that the buyer in model (II) – one monopoly firm – listened from news that the prices of overall cars for different kinds, will be changed in the next month. So, how the buyer will behave now?

So in general, it's clear that, the buyer will decide to buy now (or during this month) before tomorrow if the future prices of cars will increase. At the same time he will not buy now or during this time if the future prices for all cars will decrease. The question now: Does the buyer still have the same opportunity for the same car that has optimal rational choice as above or he will change his rational decision from that car?

The answer of the above important question is as follows: the buyer will do similar calculations as above, but he should add the time as a factor effect for making decision for which car he will buy from firm. The time strategy has actually its effect for products that are costly, like cars in our models. However, time has no effect when the product are not enough costly.



The buyer will add another strategy as time strategy  $S_{it}$  in the column of strategies corresponding to probability  $P_i$  for each  $car_i$  :  
and the previous table will change to become as follows:

The product	Buyer Calculations (Approximations)					Seller Price	Difference
	Motivations Or Shortcomings	Reward (Value)	Probabilities	Costs	Total (Costs)	Actual Price	(Payoff/Loss)
Car 1	S11	R11	P'11	R11P'11	$C'_1$	$P_1$	$\mathbb{I}'_1 = C'_1 - P_1$
	S12	R12	P'12	R12P'12			
	S13	R13	P'13	R13P'13			
	⋮	⋮	⋮	⋮			
	S1n	R1n	P'1n	R1nP'1n			
	S1t	R1t	P'1t	R1tP'1t			
Car 2	S21	R21	P'21	R21P'21	$C'_2$	$P_2$	$\mathbb{I}'_2 = C'_2 - P_2$
	S22	R22	P'22	R22P'22			
	S23	R23	P'23	R23P'23			
	⋮	⋮	⋮	⋮			
	S2n	R2n	P'2n	R2nP'2n			
	S2t	R2t	P'2t	R2tP'2t			
⋮	⋮	⋮	⋮	⋮	⋮	⋮	
Car m	Sm1	Rm1	P'm1	Rm1P'm1	$C'_m$	$P_m$	$\mathbb{I}'_m = C'_m - P_m$
	Sm2	Rm2	P'm2	Rm2P'm2			
	Sm3	Rm3	P'm3	Rm3P'm3			
	⋮	⋮	⋮	⋮			
	Smn	Rmn	P'mn	RmnP'mn			
	Smt	Rmt	P'mt	RmtP'mt			

As we know before adding time strategy:

$$\sum_{j=1}^n P_{ij} = 1 \quad \text{For every } i = 1, 2, 3, \dots, m$$

Now, after adding time strategy we have:

$$P'_{it} \geq 0 \quad \text{For all } i = 1, 2, 3, \dots, m$$

Thus:

$$P'_{ij} \leq P_{ij}$$

For all  $i = 1, 2, 3, \dots, m$

And  $j = 1, 2, 3, \dots, n$

So:

$$\sum_{j=1}^n P'_{ij} \leq \sum_{j=1}^n P_{ij} = 1$$

Now, the additional cost due to the time strategy will be:

$$R_{it} P'_{it} \geq 0 \quad \text{for all } i = 1, 2, 3, \dots, m$$

So, the buyer will reduce probabilities of buying each car for all car strategies because the effects of time strategy. Time probability for each car is a result of the reduction amount for these probabilities. This happens for all cars.

\* The cost for each car before adding time strategy as we know is:

$$C_i = \sum_{j=1}^n R_{ij} P_{ij} \quad i = 1, 2, 3, \dots, m$$

This cost  $C_i$  means that I have the ability to accept buying car  $i$  with at most this cost without time effect, otherwise I will not buy this car.

\* The new costs for each car after adding the time strategy will be:

$$C'_i = \left[ \sum_{j=1}^n R_{ij} P'_{ij} \right] + R_{it} P'_{it}$$

$$i = 1, 2, 3, \dots, m$$

This cost  $C'_i$  means that I have the ability to accept buying *car*  $i$  with at most this cost within time effect; otherwise I will not buy this car.

To answer the first question it's enough to answer the following question:

Which is larger:  $C_i$  or  $C'_i$  ?

It's clear that:

$$\sum_{j=1}^n R_{ij} P'_{ij} \leq \sum_{j=1}^n R_{ij} P_{ij}$$

Assuming that  $R_{ij}$  is still fixed for all  $i$  and  $j$ .

And since:  $R_{it} P'_{it} \geq 0$  for all  $i = 1, 2, 3, \dots, m$

We can suppose that:

$$\in_{it} = \sum_{j=1}^n R_{ij} P_{ij} - \sum_{j=1}^n R_{ij} P'_{ij}$$

$\in_{it}$  : is the reduction cost for each  $car_i$  from the previous cost (without time news) due to the time effect:

It's clear also that:  $\in_{it} \geq 0$  for all  $i = 1, 2, 3, \dots, m$

To see if the buyer derives from his optimal behavioral, we need to proceed in his new method of approximation within time strategy, in order to know what happen to the material payoff for each car.

Now we are interested in calculating the difference costs:

$$\nabla_i = C_i - C'_i = \in_{it} - R_{it} P'_{it}$$

Note that:

$$\in_{it} \geq 0 \quad \text{and} \quad R_{it} P'_{it} \geq 0$$

$$i = 1, 2, 3, \dots, m$$

Where:

- $C'_i$  : Total cost for each car after adding the time strategy.
- $C_i$  : Total cost for each car before adding time strategy.
- $\in_{it}$  : is the reduction of the total cost in  $car_i$  due to the effect of time.
- $R_{it} P'_{it}$  : is the additional cost that the buyer will add to the total cost of  $car_i$  due to the effect of time.

The following page explains three different cases for the buyer behavioral within time strategy, and it explains the result effect when one type of bounded rationality is applied.

So we have three cases:

1. If  $\in_{it} > R_{it} P'_{it}$  for  $car_i$ , then  $C_i > C'_i$

In this case, the buyer estimation cost for  $car_i$  within time, will be smaller than the cost without time effect by a positive value  $\nabla_i$ .

Thus, the payoff of this car will be reduced by  $\nabla_i$  - the different cost affected by the time for  $car_i$  - as follows:

$$\pi'_i = C'_i - P_k$$

Since  $C'_i$  is decreased, then  $\pi'_i$  will also decrease enough.

So, if  $\pi'_i$  becomes negative, then the buyer will not decide to buy this car, otherwise -  $\pi'_i$  is still positive- then the payoff of this car ( $\pi'_i$ ) will enter into competitive choice of the other payoffs for other cars, to select the global one as what happen in the previous case (no time effect).

So  $\pi_i > \pi'_i$  for car i and:

$$\pi'_i = \pi_i - \nabla_i$$

This means that the buyer payoff for this car is reduced by  $\nabla_i$ , so, the time is a negative indicator for this car. Thus, this car will have smaller opportunity to win with maximum payoff.

What we are saying now is that, in this case, the total payoff of this car is reduced -regardless by how much- so if this car was won in previous case (no time effect) with global material of payoff between all other cars, then this car may lose the opportunity to won now, since its payoff is reduced.

If this happened, then the buyer will not decide to buy the same car because it does not maximize his optimal utility or his payoff

within the time effect. Thus bounded rationality is applied in this case and it may change the best behavior for the buyer by depending on rational approach.

What happens in this case is when the price of this car will decrease in the next month. Thus, the buyer does not prefer to buy this car now or during this month. The following case tells us when  $\nabla_i$  becomes negative.

2. If  $\epsilon_{it} < R_{it} P'_{it}$  for  $car_i$ , then  $C_i < C'_i$

In this case, the buyer estimation cost for  $car_i$  within time, will be larger than the cost without time effect by absolute value  $\nabla_i$ . Thus, the payoff of this car will increase by the different cost  $\nabla_i$  affected by the time as follows:

$$\pi'_i = C'_i - P_k$$

Since  $C'_i$  is increased, and then  $\pi'_i$  will also increase. And in this case the  $car_i$  will have more opportunity to become the car that has global payoff among all cars.

So  $\pi'_i > \pi_i$  for  $car_i$  and:

$$\pi'_i = \pi_i + \nabla_i$$

This means that the buyer prefers to buy the  $car_i$  now or during this month. In this case, the opportunity of this car will increase to win with maximum payoff among all cars.

If such car did not win in the previous case (no time effect), and since now it's material payoff is increased, then its probably to win with maximum payoff, and in this case bounded rationality gives other cars the opportunity to win instead of the global one. (Optimal behavioral with perfect rationality to buy the global car) this will happen by changing the optimal behavioral for the buyer in the previous case with another one that has less rationality.

This case will happen only when the price of this car will increase during the next month. So I prefer to buy it now.

3. If  $\epsilon_{it} = R_{it} P'_{it}$  for  $car_i$ , then  $C_i = C'_i$

In this case, the buyer estimation cost for  $car_i$  within time, will be the same as the cost without time. Thus, the payoff of this car will stay without change.

$$\pi'_i = \pi_i$$

In this case, the  $car_i$  will have same payoff. This means that the buyer can not distinguish between the effect of time and the case which has no time effect. Or the buyer sees that time does not affect his global payoff for choosing his preference car even within increasing future prices of cars i.e. he does not take care about the time effect. This



case will also happen if there is no change in the price of this car during the next month i.e. the price of this car is still the same during this month and the following one.

So in both cases, future expected price will change the optimality of the human behavior by rational way according to the buyer, but at the same time, it changes the optimal behavior of the buyer. But actually, the buyer does that during this month and he changes his optimal one that he did before.

Of course, the buyer derives from his optimality by rational direction. But in conclusion, the buyer derives from his optimality using his approximation. So bounded rationality in this case are concluded.

On the other hand, if overall future prices do not change, then the optimal behavior will stay constant without changing and the buyer calculation will not differ from previous one.

## **Chapter Five: Conclusions and Suggestions**

- 5.1 Conclusions and Suggestions
- 5.2 References

## **Conclusions and Suggestions**

The result for the first model is solving the problem that is constructed in our mind by applying a Rule of – Thumb approach that Ellison suggested when we entered a super market in order to decide to buy or not to buy new product that we never have seen before. Moreover, we derived the optimal choice during the three rational models.

Firms are full rational, but firms within competitive market can be used B.R to maintain their effectiveness in the market. Firms will sacrifice by small part from their profit in order to keep their business in the market and in order to attract customers to buy their products. Firms do that by adding some prizes and facilities to their products especially to those products that have small selling share.

In case of monopoly firm, it does not take care to do bounded rationality, especially when its products have the same selling share, because it thinks such bounded rationality will be identified as an additional cost to the firm and it can achieve the same profits without losing anything. However, if the firm's products have different selling share and the firm need to sell the product that have small enough selling share, then the firm have two choices to do: The first one - which is easy for firm - is to change the prices since there is no competition here. The second one is doing bounded rationality by sacrificing small amount of the firm's profit for these products by adding new services in order to encourage buyers to buy these products. But in this case firms force consumers to do B.R using their full rational approach of calculation as we explained before.

The term bounded rationality is used to designate rational choice that takes into account the cognitive limitations of both knowledge and cognitive capacity. Bounded rationality is a central theme in behavioral economics. It is concerned with the ways in which the actual decision-making process influences decisions.

However, since the human behavior may face irrationality in their approximations or decisions - especially in game theory problems, due to fairness, cooperation and altruism - then bounded rationality is a result for such irrationality. But in most cases, the derivation of human behavioral is done by full rationality, especially in economics' issues, and bounded rationality in such issues is weaker than the previous one.

The question now, why there is no bounded rationality models until this moment. Is there some fundamental thing that prevents us from constructing useful bounded rationality models? The answer for this important question opens new area for the researchers to do thesis in a way to solve this problem. I am one of whom will continue in this field of study.

## References

- Andreoni, James, and Miller, John H., "Rational Cooperation in the Finitely Repeated Prisoner's Dilemma: Experimental Evidence," *The Economic Journal*, May 1993, 103(418), 570-585.
- Armstrong, M. 2006 .*Recent Developments in the Economics of Price Discrimination*, in *Advances in Economics and Econometrics: Theory and Applications: Ninth World Congress of the Econometric Society*, ed. by R. Blundell, W. Newey, and T. Persson. Cambridge University Press, Cambridge, UK.
- Ellison, G .2006 .*Bounded Rationality in Industrial Organization*. MIT working paper.
- Fudenberg, Drew and David Kreps (1988), "A Theory of Learning, Experimentation and Equilibrium in Games", mimeo.
- Fudenberg, D. and Tirole, J. 1996. *Game Theory*. London: The MIT Press.
- Fudenberg, D, and Tirole, J. 2000 .*Customer Poaching and Brand Switching*.RAND Journal of Economics, 31, 634-657.
- Joskow, Paul (1973), "Pricing Decisions of Regulated Firms: A Behavioral Approach," *The Bell Journal of Economics and Management Strategy*, 4, 118-140.

Kahneman, Daniel, Jack L. Knetsch, and Richard Thaler (1986),  
“*Fairness as a Constraint on Profit Seeking: Entitlements in the Market,*”  
*American Economic Review*, 76, 728-741.

Kreps, David, Robert Wilson, Paul Milgrom, and John Roberts. “Rational  
Cooperation in the Finitely Repeated Prisoners’ Dilemma.” *Journal of  
Economic Theory* 27, no. 2 (August 1982): 245–252.

Möbius, Markus (2001), “*Death Through Success: The Rise and Fall of  
Local Service Competition at the Turn of the Century,*” mimeo.

Rabin, M. 1993. *Incorporating Fairness into Game Theory and  
Economics*. *American Economic Review*, LXXXIII, 1281-1303.

Rubinstein, A. 1998. “*Modeling Bounded Rationality*”. London: The MIT  
Press.

Simon, H. 1955. “A Behavioral Model of Rational Choice”. *The Quarterly  
Journal of Economics*, 99-118.

Tanga McDaniel, E. Elisabet Rutström and Melonie Williams.  
“*Incorporating Fairness Into Game Theory and Economics: Comment,*”  
University of South Carolina. *The Economic Journal*, March 1994.

Tirole, J. 1988. *The Theory of Industrial Organization*. London: The MIT  
Press.